International Journal of Theoretical Physics, Vol. 5, No. 3 (1972), pp. 151–159

Current Density in Relativistic Quantum Mechanics

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Received: 23 *April* 1971

Abstract

We extend to relativistic theories the concepts of probability density and probability current density of nonrelativistic quantum mechanics, together with the charge and current densities that are used as sources of the electromagnetic field in the semi-classical theory of radiation. There are some limitations in the procedure, especially in the case of several particles.

1. *Introduction*

The wave function in nonrelativistic quantum mechanics is usually interpreted as a probability amplitude. From it we obtain not only the probability density, but also a probability current density. When these are multiplied by the charge of the particle, they are used as sources of the electromagnetic field in the semi-classical theory of radiation.

We have extended the idea of probability amplitudes to the relativistic quantum mechanics of scalar particles (Marx, 1969, 1970a) and of spin-$\frac{1}{2}$ particles (Marx, 1970b, c). We now propose to examine in greater detail the definition of similar densities in these theories, based on the Klein-Gordon equation and the modified Dirac equation for free particles and particles in an external electromagnetic field.

We first come to the conclusion that the charge and current densities obtained from Noether's theorem do not lead to a reasonable interpretation in terms of probability densities. Since conservation laws refer primarily to integrated quantities, such as the total charge, we have considerable leeway in the definition of densities. We study two other possibilities. One is the straightforward generalization of the nonrelativistic definitions, but these densities do not obey a differential conservation law. By a slight modification, we can eliminate this difficulty for free fields, but it reappears when we consider the interaction. In that case, we have nonlocal effects, which we consider an acceptable consequence of the use of integral operators in the Hamiltonian.

The difficulties are more severe when we consider the theory of several noninteracting particles in an external electromagnetic field. We find no reasonable expression for the probability current density, and this problem can be traced to the different modes of propagation of particles and antiparticles. We do find a current density that can be used as a source for a dynamical electromagnetic field in a semi-classical theory of radiation, but there is a lack of motivation that serves to emphasize the approximate nature of this approach, especially in a relativistic theory.

We recall the nonrelativistic theory in Section 2, in order to compare it with the relativistic generalizations. These are presented in Section 3 for scalar particles and in Section 4 for spin-$\frac{1}{2}$ particles. We briefly discuss the theory for several particles in Section 5, and conclude with some remarks in Section 6.

We use natural units, the time-favoring metric in space-time and other conventions explained in our earlier papers on the subject.

2. *The Nonrelativistic Theory*

We can obtain the nonrelativistic Schrödinger equation for free particles

$$i\dot{\psi} = -(1/2m)\nabla^2\psi \tag{2.1}$$

from the Lagrangian density

$$\mathscr{L}_0 = \tfrac{1}{2}i(\psi^*\dot{\psi} - \dot{\psi}^*\psi) - (1/2m)(\nabla\psi^*).\nabla\psi \tag{2.2}$$

Noether's theorem applied to gauge transformations of the first kind then gives the probability density

$$\rho = |\psi|^2 \tag{2.3}$$

and the probability current density

$$\mathbf{j} = (1/2m)[\psi^*(-i\nabla\psi) + (i\nabla\psi^*)\psi] \tag{2.4}$$

which satisfy the differential conservation law

$$\dot{\rho} + \nabla.\mathbf{j} = 0 \tag{2.5}$$

We can introduce the interaction with an external electromagnetic field by means of the gauge invariant substitution

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu \tag{2.6}$$

to obtain the Lagrangian density

$$\begin{aligned}\mathscr{L} = \tfrac{1}{2}i(\psi^*\dot{\psi} - \dot{\psi}^*\psi) - qA_0\psi^*\psi \\ - (1/2m)[(\nabla - iq\mathbf{A})\psi]^*.(\nabla - iq\mathbf{A})\psi\end{aligned} \tag{2.7}$$

The Schrödinger equation becomes

$$i\dot{\psi} = [-(1/2m)(\nabla - iq\mathbf{A})^2 + qA_0]\psi \tag{2.8}$$

and Noether's theorem gives the same probability density (2.3) and the probability current density

$$\mathbf{j} = (1/2m)[\psi^*(-i\nabla - q\mathbf{A})\psi + \psi(i\nabla - q\mathbf{A})\psi^*] \tag{2.9}$$

which still satisfy the conservation law (2.5).

When these densities are multiplied by q, they are used in the semi-classical theory of radiation as a source of the (classical) electromagnetic field. The underlying assumption is that the particle itself, not only the probability, is spread in space with density ρ. This is contrary to the basic probabilistic interpretation of quantum mechanics, but is considered to be a good approximation in certain problems.

The complete interaction of the two classical fields can then be obtained from the Lagrangian density

$$\mathscr{L}' = \mathscr{L} - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{2.10}$$

where the field A_μ in $\mathscr{L}$ can be replaced by $A_\mu + A_\mu^{\text{ext}}$ to consider both a dynamical and an external electromagnetic field. The equations of motion for the interacting fields are nonlinear, and they are usually solved by means of a perturbation expansion. There is radiation reaction when we consider higher than lowest-order terms or a non-perturbative solution, which would be especially interesting in the case of bound states.†

3. *Scalar Particles*

We have shown (Marx, 1969, 1970a) how the probabilistic interpretation of nonrelativistic quantum mechanics can be extended to the relativistic theory of charged scalar particles in an external electromagnetic field, based on the Klein-Gordon equation. The Lagrangian density for free particles is

$$\mathscr{L}_0 = \phi^*_{,\mu}\phi_{,\mu} - m^2\phi^*\phi \tag{3.1}$$

from which we obtain the equation

$$(\partial^2 + m^2)\phi = 0 \tag{3.2}$$

and the conserved current density four-vector

$$j_\mu = ie(\phi^*\phi_{,\mu} - \phi^*_{,\mu}\phi) \tag{3.3}$$

for particles of charge $+e$. The probability amplitudes for the particle and the antiparticle are defined by

$$g^{(\pm)}(x) = \tfrac{1}{2}(1 \pm i\tilde{E}^{-1}\partial_0)\phi(x) \tag{3.4}$$

where we use the integral operator

$$\tilde{E} = (-\nabla^2 + m^2)^{1/2} \tag{3.5}$$

In terms of these amplitudes, equation (3.2) takes the form of two Schrödinger-type equations

$$i\dot{g}^{(\kappa)} = \kappa\tilde{E}g^{(\kappa)}, \qquad \kappa = \pm \tag{3.6}$$

† Radiation reaction for classical particles is discussed in detail by Rohrlich (1965). Other problems in nonlinear theories of quantum mechanics are discussed by Leiter (1969, 1970).

and the charge density becomes

$$j_0 = \tfrac{1}{2}e \sum_{\kappa} [\kappa(\tilde{E}^{1/2} g^{(\kappa)*}) \tilde{E}^{-1/2} g^{(\kappa)} + \kappa(\tilde{E}^{-1/2} g^{(\kappa)*}) \tilde{E}^{1/2} g^{(\kappa)} + (\tilde{E}^{1/2} g^{(\kappa)*}) \tilde{E}^{-1/2} g^{(-\kappa)} - (\tilde{E}^{-1/2} g^{(\kappa)*}) \tilde{E}^{1/2} g^{(-\kappa)}] \tag{3.7}$$

It is hard to find an interpretation for the last two terms, which represent some kind of interference between particle and antiparticle amplitudes, unlikely to have a physical meaning in the absence of an interaction. A similar difficulty occurs in the separation of the orbital and spin parts of the angular momentum of spin-$\frac{1}{2}$ particles. The separation most commonly found in the literature does not give two separately conserved parts, but we have shown (Marx, 1968) how a reasonable use of the probability amplitudes gives a better answer to this problem. We observe that the total charge

$$Q = \int j_0 \, d^3 x \tag{3.8}$$

can be rewritten in the form†

$$Q = e \int (\rho^{(+)} - \rho^{(-)}) \, d^3 x \tag{3.9}$$

where we introduce the probability densities

$$\rho^{(\pm)} = |g^{(\pm)}|^2 \tag{3.10}$$

It is then obvious that we should consider

$$j_0' = e(\rho^{(+)} - \rho^{(-)}) \tag{3.11}$$

for the charge density, and the relativistic velocity operator suggests that the current density should be

$$\mathbf{j}' = \tfrac{1}{2}e \sum_{\kappa} [g^{(\kappa)*}(-i\nabla/\tilde{E}) g^{(\kappa)} + g^{(\kappa)}(i\nabla/\tilde{E}) g^{(\kappa)*}] \tag{3.12}$$

A direct calculation using the equations of motion (3.6) shows that these densities do not obey a differential conservation law, which we find objectionable in the case of free particles.

We further note that the last two terms in equation (3.7) do not contribute to the total charge, which suggests that we try

$$j_0'' = \tfrac{1}{2}e \sum_{\kappa} [\kappa(\tilde{E}^{1/2} g^{(\kappa)*}) \tilde{E}^{-1/2} g^{(\kappa)} + \kappa(\tilde{E}^{-1/2} g^{(\kappa)*}) \tilde{E}^{1/2} g^{(\kappa)}] \tag{3.13}$$

$$\mathbf{j}'' = \tfrac{1}{2}e \sum_{\kappa} [(\tilde{E}^{-1/2} g^{(\kappa)*})(-i\nabla) \tilde{E}^{-1/2} g^{(\kappa)} + (\tilde{E}^{-1/2} g^{(\kappa)}) i\nabla \tilde{E}^{-1/2} g^{(\kappa)*}] \tag{3.14}$$

and we find that these densities do obey the conservation law. The contributions from particles and antiparticles are separated, and actually both are conserved in the absence of interactions. We thus find this definition more acceptable in terms of probability amplitudes and probability densities, which can be obtained by dividing j_μ'' by e. One might expect offhand that

† This equation shows that the total charge of the system is not necessarily equal to the charge of the particle.

the contribution of particles and antiparticles to the current density would have opposite signs; the reason this is not so can be traced to the minus sign in equation (3.6) for $g^{(-)}$, which is related to the concept of antiparticles propagating backward in time.

We obtain the correct nonrelativistic limit when neglect $g^{(-)}$ and replace $\tilde{E}$ by m in equations (3.13) and (3.14), but this is also true for the other alternatives.

When an external electromagnetic field is present, we have to modify the equations accordingly to

$$\mathscr{L} = (D_\mu{}^* \phi^*) D_\mu \phi - m^2 \phi^* \phi \tag{3.15}$$

$$(D^2 + m^2)\phi = 0 \tag{3.16}$$

$$j_\mu = ie[\phi^* D_\mu \phi - (D_\mu{}^* \phi^*)\phi] \equiv ie\phi^* \overleftrightarrow{D}_\mu \phi \tag{3.17}$$

$$g^{(\pm)}(x) = \tfrac{1}{2}(1 \pm i\tilde{E}^{-1} D_0)\phi(x) \tag{3.18}$$

$$\begin{aligned} i\dot{g}^{(\kappa)} = {} & [\kappa\tilde{E} + \tfrac{1}{2}e(\tilde{E}^{1/2} A_0 \tilde{E}^{-1/2} + \tilde{E}^{-1/2} A_0 \tilde{E}^{1/2}) \\ & + \tfrac{1}{2}\kappa e\tilde{E}^{-1/2}(i\nabla . \mathbf{A} + i\mathbf{A} . \nabla + e\mathbf{A}^2)\tilde{E}^{-1/2}]g^{(\kappa)} \\ & + [\tfrac{1}{2}e(\tilde{E}^{1/2} A_0 \tilde{E}^{-1/2} - \tilde{E}^{-1/2} A_0 \tilde{E}^{1/2}) \\ & + \tfrac{1}{2}\kappa e\tilde{E}^{-1/2}(i\nabla . \mathbf{A} + i\mathbf{A} . \nabla + e\mathbf{A}^2)\tilde{E}^{-1/2}]g^{(-\kappa)} \end{aligned} \tag{3.19}$$

Equations (3.8) and (3.9) are still valid, so that we can retain equation (3.13) for the charge density, while replacing equation (3.14) by

$$\mathbf{j}'' = \tfrac{1}{2}e \sum_\kappa [(\tilde{E}^{-1/2} g^{(\kappa)*})(-i\mathbf{D})\tilde{E}^{-1/2} g^{(\kappa)} + (\tilde{E}^{-1/2} g^{(\kappa)}) i\mathbf{D}^* \tilde{E}^{-1/2} g^{(\kappa)*}] \tag{3.20}$$

which still has the right nonrelativistic limit. A direct computation using equation (3.19) shows that the conservation law (2.5) is not satisfied. This shows that we cannot use j_μ'' as a source, for the electromagnetic field, since Maxwell's equations

$$F_{\mu\nu,\nu} = j_\mu \tag{3.21}$$

imply that the differential conservation law must be satisfied. We are thus inclined to consider j_μ'', without the overall factor e, a probability current density. As we have pointed out already for the nonrelativistic case, the probabilistic interpretation of quantum mechanics in no way leads to the proportionality between the probability density and the charge density. These arguments are thus not a reason to reject j_μ as defined in equation (3.17) as a source of the electromagnetic field, which can be included in the theory through the Lagrangian density (2.10).

The lack of a differential conservation law for the probability current density would indicate that a change in the probability of finding the particle or the antiparticle in a finite volume is not related to a flux across its surface. These nonlocal effects should be expected in a relativistic theory that uses integral operators such as $\tilde{E}$. Furthermore, we have pointed out (Marx, 1970d) that probability amplitudes should not be interpreted in the usual sense at intermediate times. If we start with a particle at the initial time, it will either show up as a particle at the final time or as an antiparticle at the initial time. The imposition of the condition that the antiparticle amplitude be zero at the final time indicates that no observations should be permitted at intermediate times, since they would affect the system and the boundary conditions. These arguments apply to particle scattering and pair annihilation, but similar ones can be stated for antiparticle scattering and pair creation.

4. *Spin-½ Particles*

The Dirac equation does not lend itself to a formulation of relativistic quantum mechanics in terms of probability amplitudes for a single particle, due to the positive definite nature of what is usually interpreted as a probability density. In a relativistic theory it is not probability but charge that is conserved. We do not find the hole theory of positrons and the infinite sea of filled negative-energy electron states a satisfactory correlation between the Dirac theory and physical reality. We have proposed instead a theory based on the Klein-Gordon equation for two-component spinors (Marx, 1970c) and another based on a modification of the Dirac equation (Marx, 1970b). The latter is the one that best describes electrons.

We start from the Dirac equation for free particles,

$$(-i\gamma.\partial + m)\psi = 0 \tag{4.1}$$

which can be obtained from the Lagrangian density

$$\mathscr{L}_0 = \tfrac{1}{2}i(\bar{\psi}\gamma_\mu\psi_{,\mu} - \bar{\psi}_{,\mu}\gamma_\mu\psi) - m\bar{\psi}\psi \tag{4.2}$$

and Noether's theorem gives the conserved current density

$$j_\mu = -e\bar{\psi}\gamma_\mu\psi \tag{4.3}$$

for particles of charge $-e$. The probability amplitudes are obtained from the four-component spinor ψ by a Foldy-Wouthuysen transformation

$$g(x) = \left(\frac{\tilde{E}+m}{2\tilde{E}}\right)^{1/2}\left(1 - \frac{i\boldsymbol{\gamma}.\boldsymbol{\nabla}}{\tilde{E}+m}\right)\psi(x) \tag{4.4}$$

and they obey equations of the form (3.6). We introduce the electromagnetic interaction through the gauge invariant substitution (2.6), but then we modify the Dirac equation to obtain the equations of motion

$$\begin{aligned} i\dot{g}^{(\kappa)} = \Bigg\{\kappa\tilde{E} - \mathrm{e}\left(\frac{\tilde{E}+m}{2\tilde{E}}\right)^{1/2}&\Bigg[\left(A_0 - \frac{\boldsymbol{\sigma}.\boldsymbol{\nabla}}{\tilde{E}+m}A_0\frac{\boldsymbol{\sigma}.\boldsymbol{\nabla}}{\tilde{E}+m}\right) \\ &+ i\kappa\left(\frac{\boldsymbol{\sigma}.\boldsymbol{\nabla}}{\tilde{E}+m}\boldsymbol{\sigma}.\mathbf{A} + \boldsymbol{\sigma}.\mathbf{A}\frac{\boldsymbol{\sigma}.\boldsymbol{\nabla}}{\tilde{E}+m}\right)\Bigg]\left(\frac{\tilde{E}+m}{2\tilde{E}}\right)^{1/2}\Bigg\}g^{(\kappa)} \\ &- e\left(\frac{\tilde{E}+m}{2\tilde{E}}\right)^{\frac{1}{2}}\Bigg[i\left(A_0\frac{\boldsymbol{\sigma}.\boldsymbol{\nabla}}{\tilde{E}+m} - \frac{\boldsymbol{\sigma}.\boldsymbol{\nabla}}{\tilde{E}+m}A_0\right) \\ &- \kappa\left(\boldsymbol{\sigma}.\mathbf{A} + \frac{\boldsymbol{\sigma}.\boldsymbol{\nabla}}{\tilde{E}+m}\boldsymbol{\sigma}.\mathbf{A}\frac{\boldsymbol{\sigma}.\boldsymbol{\nabla}}{\tilde{E}+m}\right)\Bigg]\left(\frac{\tilde{E}+m}{2\tilde{E}}\right)^{1/2}g^{(-\kappa)} \end{aligned} \tag{4.5}$$

The current density (4.3) is conserved for the Dirac equation, but not when the equations of motion (4.5) are used. Instead, the densities

$$j_0'' = -\tfrac{1}{2}e \sum_\kappa [\kappa(\tilde{E}^{1/2} g^{(\kappa)})^\dagger \tilde{E}^{-1/2} g^{(\kappa)} + \kappa(\tilde{E}^{-1/2} g^{(\kappa)})^\dagger \tilde{E}^{1/2} g^{(\kappa)}] \quad (4.6)$$

$$\begin{aligned} \mathbf{j}'' = -\tfrac{1}{2}e \sum_\kappa [&(-i\sigma\boldsymbol{\sigma}.\mathbf{D}\tilde{E}^{-1/2} g^{(\kappa)})^\dagger \tilde{E}^{-1/2} g^{(\kappa)} \\ &+ (\tilde{E}^{-1/2} g^{(\kappa)})^\dagger (-i\sigma\boldsymbol{\sigma}.\mathbf{D}\tilde{E}^{-1/2} g^{(\kappa)})] \end{aligned} \quad (4.7)$$

obey a conservation law for free particles, while for particles in an external electromagnetic field they do not, although the total charge

$$Q = \int j_0'' \, d^3 x \quad (4.8)$$

is conserved. This is no longer true for the charge obtained from j_0. We have chosen this particular form of the current density (4.7) by analogy to the terms in the Hamiltonian; the relation

$$\sigma\boldsymbol{\sigma}.\mathbf{p} = \mathbf{p} + i\mathbf{p} \wedge \boldsymbol{\sigma} \quad (4.9)$$

indicates that the spin of the particle contributes to the current density.

We have now a situation that does not differ significantly from that discussed in Section 3 for scalar particle, and the same general comments apply.†

5. *Many Particles*

The generalization of nonrelativistic quantum mechanics to several particles involves a wave function $\psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n; t)$, which obeys the equation

$$i\dot{\psi} = \sum_{k=1}^{n} [-(1/2m_k)\mathbf{D}_k^2 + q_k A_0(\mathbf{x}_k, t)]\psi \quad (5.1)$$

We can then generalize equations (2.3) and (2.9) to

$$j_0(\mathbf{x}, t) = \sum_{k=1}^{n} q_k \int \delta(\mathbf{x} - \mathbf{x}_k) |\psi|^2 d^3 x_1 \ldots d^3 x_n \quad (5.2)$$

$$\mathbf{j}(\mathbf{x}, t) = -i \sum_{k=1}^{n} (q_k/2m_k) \int \delta(\mathbf{x} - \mathbf{x}_k)(\psi^* \overleftrightarrow{\mathbf{D}}_k \psi) d^3 x_1 \ldots d^3 x_n \quad (5.3)$$

and these densities satisfy the conservation law (2.5). When the particles are identical, the wave functions are symmetric or antisymmetric and the densities take somewhat simpler forms.

We have discussed the relativistic quantum mechanics of identical particles (Marx, 1970a, c) within the framework of Dirac's many-time

† The idea of a position of the charge as different from the position of the particle (Schröder, 1964) does not appear to be helpful for our understanding of relativistic quantum mechanics.

formalism. For distinguishable scalar particles,† the probability amplitudes $g^{(\kappa_1\ldots\kappa_n)}(x_1,\ldots,x_n)$ obey equations of the form

$$i\partial_{j0}\, g^{(\kappa_1\ldots\kappa_j\ldots\kappa_n)} = \sum_{\kappa_j'} H_{j\kappa_j\kappa_j'}\, g^{(\kappa_1\ldots\kappa_j'\ldots\kappa_n)} \tag{5.4}$$

where the Hamiltonian operators $H_{j\kappa_j\kappa_j'}$ for the jth particle can easily be determined from equation (3.19). The conserved charge (3.9) can be generalized to

$$Q = \sum_{\{\kappa_j\}} \int q_1 \ldots q_n |g^{(\kappa_1\ldots\kappa_n)}(\mathbf{x}_1, t; \ldots; \mathbf{x}_n, t)|^2 d^3x_1 \ldots d^3x_n \tag{5.5}$$

We do not find a reasonable interpretation for the products of the charges that we have in equation (5.5). On the other hand, if we define the charge of particle 1 as

$$Q_1 = q_1 \sum_{\{\kappa_j\}} \int \kappa_1 |g^{(\kappa_1\ldots\kappa_n)}(\mathbf{x}_1, t; \ldots; \mathbf{x}_n, t)|^2 d^3x_1 \ldots d^3x_n \tag{5.6}$$

we find that it is not conserved. Again this difficulty is not surprising since conservation of charge in our theory relates probabilities obtained from amplitudes in which particle times are t_i and antiparticle times are t_f with those in which they are t_f and t_i respectively. That is, any of the particles that is given at t_i can be either scattered or annihilated, and any of the antiparticles given at t_f can have been either scattered or created. No special meaning is found for these amplitudes when we set all times equal to t.

Thus, we come to the conclusion that there are no reasonable generalization for the probability current densities.

We now proceed to generalize $j_\mu(x)$, as defined in equation (3.17), in terms of the wave function $\phi(x_1,\ldots,x_n)$. We first define

$$j_{\mu\nu\cdots\lambda}(x_1, x_2, \ldots, x_n) = i^n \phi^* \overleftrightarrow{D}_\mu(x_1)\, \overleftrightarrow{D}_\nu(x_2) \ldots \overleftrightarrow{D}_\lambda(x_n)\, \phi \tag{5.7}$$

which obeys n separate conservation laws. We then set

$$j_\mu(x) = \sum_{k=1}^{n} q_k \int \delta(\mathbf{x} - \mathbf{x}_k) j_{0\cdots\mu\cdots0}(x_1, \ldots, \mathbf{x}_k, t, \ldots, x_n)\, d^3x_1 \ldots d^3x_n \tag{5.8}$$

and the conservation laws show that it is independent of times other than t. But we do not have a compelling reason to justify this definition, except that it gives a conserved current density that can be used as a source for the electromagnetic field.

We find no additional difficulties when the particles are identical.

6. *Concluding Remarks*

We have examined in detail the ideas of probability densities and probability current densities in relativistic quantum mechanics, especially

† If we wish to consider spin-$\frac{1}{2}$ particles, all we have to do is add the necessary spin indices to the amplitudes.

in relation to the charge and current densities that serve as a source of the electromagnetic field in the semi-classical theory of radiation.

We observe that this relation is in the nature of an approximation rather than of fundamental significance in the nonrelativistic theory. We find that the most appropriate definitions for these two sets of densities differ considerably in the relativistic theory of charged scalar particles, but that this does not imply any serious inconsistency. Similar conclusions are reached for the theory of spin-$\frac{1}{2}$ particles.

The generalization of probability currents to a theory of many particles was not successful, although a charged current density was found. This leads once more to the conclusion that certain amplitudes where the times are either t_i or t_f can be interpreted properly as probability amplitudes, while no observations are allowed at intermediate times.

More extensive studies of the complete interactions between the particle fields and the electromagnetic field should show more clearly to what extent relativistic quantum mechanics and a semi-classical theory of radiation explain the results of experiments, and what types of problems require a solution within the framework of a quantum theory of fields.

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